(1) Prove that a continuous function on \( \mathbb{R} \) has a finite or countable number of strict local maxima.

(2) Proof or counterexample: Let \( f \) be a continuous function on \([0,1]\) that is differentiable on a dense subset. Also, \( f' > 0 \) wherever it is defined. Then \( f \) is increasing. (Hint: think about the Cantor function.)

(3) Find
\[
\lim_{n \to \infty} n^2 \int_0^1 e^{x^2} x^n (1 - x) \, dx.
\]
Hint: \( \lim_{n \to \infty} n^2 \int_0^1 x^n (1 - x) \, dx = 1. \)

(4) Let \( a_n, b_n \geq 0 \). Assume that \( \sum a_n \) converges and that \( \limsup_{n \to \infty} \frac{b_n}{a_n} \leq M < \infty \). Show that \( \sum b_n \) converges.

(5) Let \( f(x) \) be a differentiable mapping of the connected open subset \( V \) of \( \mathbb{R}^n \) to \( \mathbb{R}^m \). Suppose that \( f'(x) = 0 \) on \( V \). Prove that \( f \) is constant on \( V \).

(6) Let \( f(x, y) = (u, v) \), where \( u = x^4 - y^4 \) and \( v = 2xy \), be a map from \( \mathbb{R}^2 \) to \( \mathbb{R}^2 \). (a) Show that if \( (u, v) \neq (0,0) \) then \( f \) has an inverse in a neighborhood of \((u, v)\). (b) Show that there is no neighborhood of \((0,0)\) in which \( f \) has an inverse.