1. Let $f : (0, 1) \rightarrow \mathbb{R}$ be continuous, bounded and decreasing. Prove that $f$ is uniformly continuous on $(0, 1)$.

2. Consider the function $f : \mathbb{R}^n \rightarrow \mathbb{R}$, given by $f(x) = \sum_{j=1}^{n} \frac{x_j^3}{||x||^2}$ if $x \neq 0$, and $f(0) = 0$, where $x = (x_1, \ldots, x_n)$ and $||x||$ is the Euclidean norm of $x$. Prove that $f$ is continuous on $\mathbb{R}^n$.

3. Prove that the system
   \[
   xy^5 + yu^5 + zu^5 = 1, \\
   x^5y + y^5u + z^5v = 1,
   \]
has a unique solution $u = f(x, y, z)$, $v = g(x, y, z)$, in a neighborhood of the point $(u, v, x, y, z) = (1, 0, 0, 1, 1)$. Find $\frac{\partial u}{\partial x}(0, 1, 1)$.

4. Let $\mathbb{Q}_0$ be the set of rationals in the interval $[0, 1]$. For a bounded function $f : \mathbb{Q}_0 \rightarrow \mathbb{R}$, and $n = 1, 2, \ldots$, define
   \[
   S_n(f) = \frac{1}{n} \sum_{k=1}^{n} f(k/n).
   \]
   If $\lim_{n \to \infty} S_n(f)$ exists, we say that $f$ is $S$-summable, and let $S(f) = \lim_{n \to \infty} S_n(f)$ denote this limit. Let $f_1, f_2, \ldots$ be bounded functions on $\mathbb{Q}_0$ which are $S$-summable, and suppose that $f_k \to f$ uniformly on $\mathbb{Q}_0$ as $k \to \infty$. Prove that $f$ is $S$-summable, and that $\lim_{k \to \infty} S(f_k) = S(f)$.

5. Let $a_1, a_2, \ldots$ be a sequence of real numbers such that $\lim_{k \to \infty} a_k = L \in \mathbb{R}$ exists. For $0 < p < 1$ define
   \[
   A(p) = \sum_{k=1}^{\infty} p(1 - p)^{k-1} a_k.
   \]
   Prove that this sum converges, and that $\lim_{p \to 0} A(p) = L$.

6. Prove that
   \[
   \lim_{n \to \infty} \frac{1}{n^{5/2}} \sum_{k=1}^{n} k^{3/2} = \frac{2}{5}.
   \]