1. Use $e = 2 + \frac{1}{2!} + \frac{1}{3!} + \cdots$ to prove that $e$ is irrational.

2. Let $a_n, b_n \geq 0$, assume that $\sum a_n$ converges and that $\limsup_{n \to \infty} \frac{b_n}{a_n} \leq M < \infty$ show that $\sum b_n$ converges.

3. Let $f$ be bounded on the real interval $(a, b)$, show that if in addition $f$ is both continuous and monotone then $f$ is uniformly continuous.

4. Define $f(x) = \begin{cases} 0 & \text{, } x \text{ irrational} \\ \frac{1}{n} & \text{, } x = m/n \text{ where } m \text{ and } n \text{ relatively prime} \end{cases}$. Prove that $f$ is integrable on $[0, 1]$.

5. Let $\{f_n\}$ be a sequence of uniformly bounded Riemann integrable functions on $[0, 1]$, set $F_n(s) = \int_0^s f_n(t) \, dt$ for $0 \leq s \leq 1$. Prove that a subsequence of $\{F_n\}$ converges uniformly on $[0, 1]$.

6. Let $f(x)$ be a differentiable mapping of the connected open subset $V$ of $\mathbb{R}^n$. Suppose that $f'(x) = 0$ on $V$, prove that $f$ is constant on $V$.

7. Let $f(x, y) = (u, v)$ where $u = x^2 - y^2$ and $v = 2xy$ describe a map from $\mathbb{R}^2$ to $\mathbb{R}^2$. (a) What is the range of this map? (b) Show that if $(u, v) \neq (0, 0)$ then $f$ has an inverse in a neighborhood of $(u, v)$. (c) Show that there is no neighborhood of $(0, 0)$ in which $f$ has an inverse.