1. Let $A$ be a matrix and assume $A^2$ has characteristic polynomial $x^3(x-1)^2$ and minimal polynomial $x^2(x-1)$. What are the possible Jordan canonical forms of $A$?

2. Let $T: V \rightarrow W$ be a linear transformation between two vector spaces $V$ and $W$. Show that $T$ is injective if and only if $Ker(T) = \{ v \in V \mid T(v) = 0 \}$ only contains the vector $0$.

3. Let $T: V \rightarrow W$ be a linear transformation between two finite dimensional vector spaces $V$ and $W$. Show that $T$ is an isomorphism if and only if the dual map $T^*: W^* \rightarrow V^*$ is an isomorphism.

4. Let $T: V \rightarrow V$ be a linear operator on a vector spaces $V$ and assume $v_1, v_2, \ldots, v_k$ are eigenvectors of $T$ corresponding to the distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_k$. Show that $v_1, v_2, \ldots, v_k$ are linearly independent.

5. Suppose $A$ is an $n \times n$ matrix over the real numbers $R$. Show that $A$ is diagonalizable over $R$ if and only if we can find a basis for $R^n$ consisting of eigenvectors for $A$.

6. (a) Assume $T$ is a normal linear operator on a finite dimensional complex inner product vector space. Show that eigenvectors corresponding to distinct eigenvalues are orthogonal.

   (b) Show by example that this need not be true if $T$ is not normal.