MAT 285

Final Exam, December 14, 2005

Name ___________________________ SU ID # ___________________________

Circle your instructor’s name:

Dickerson, Gogus, Hsiang, Lindberg, Palider, Pelley, Vogel.

PLEASE READ:

1. Place all your work on these sheets. Do not add any sheets to this exam.

2. You may use a TI-83 on this exam. You may not use any other calculator nor may you use another student’s calculator.

3. Answers without supporting work/reasons will receive no credit. Label answers and give units when appropriate. Please be neat.

4. There are 10 problems (with parts) and total of 10 sheets of papers, printed only on one side, to this exam including this cover page and a blank sheet for scratch work. Be sure that you have all the pages - CHECK NOW!

5. PUT YOUR NAME ON EVERY PAGE.

PLEASE DO NOT WRITE BELOW

***** ***** ***** ***** ***** *****

<table>
<thead>
<tr>
<th>Problem number</th>
<th>Possible points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100</strong></td>
<td></td>
</tr>
</tbody>
</table>
1. A bank offers an account that pays 2.25% interest per year, compounded continuously. A customer puts $2500.00 in the account.

   (a) How much will be in the customer’s account after 2 years? Give answer to the nearest tenth of a year. [5 points]

   (b) How long will it take for the customer’s account to triple? Give answer to the nearest tenth of a year. [5 points]

2. Sales (in $) of CD’s on day x of the year is given by: \( S(x) = 100 + 55 \sin\left(\frac{\pi}{365}x - 10\right) \).

   (a) Find the period, amplitude, vertical shift (that is, midline) of \( S(x) \). [5 points]

   (b) Find the sale on day 180. [5 points]
3. The graph of the function $f(x)$ is given below. [10 points]

(a) Using the above graph, find the following limits (if a limit does not exist, then write DNE);

i. $\lim_{x \to -1^+} f(x) =$

ii. $\lim_{x \to -1} f(x) =$

iii. $\lim_{x \to 1} f(x) =$

iv. $\lim_{x \to 2} f(x) =$

(b) Circle the $x$ values where $f(x)$ is continuous: $-1, 0, 1, 2$.

(c) Circle the $x$ values where $f(x)$ is differentiable: $-1, 0, 1, 2$. 
4. For the following functions, find the indicated derivative (do not simplify):

(a) \( f(x) = x^2 - \frac{1}{x^3} + \ln(x^2) + e^{x^3-2x} \), \hspace{1cm} \text{[2.5 points]}

\[ f'(x) = \]

(b) \( h(x) = \sqrt{x^3 - \tan(2x)} \), \hspace{1cm} \text{[2.5 points]}

\[ h'(x) = \]

(c) \( w(z) = \frac{\ln(z)}{z^3 + 4} \), \hspace{1cm} \text{[2.5 points]}

\[ w'(z) = \]

(d) \( s(t) = t \cos(t) \)

[You are asked to compute the second derivative of the function \( s(t) \).] \hspace{1cm} \text{[2.5 points]}

\[ s''(t) = \]
5. The position (in meters) of a particle moving along a line is given by:

\[ s(t) = -t^3 + 3t^2 + 16, \]

where \( t \) is in minutes. [10 points]

(a) Find the velocity when \( t = 3.5 \) minutes. Give units for answer.

(b) Find the acceleration when \( t = 1.5 \) minutes. Give units for answer.
6. A rocket is rising vertically from its launching pad. An observer is positioned 3,000 feet from the launching pad. Find the rate of change of the distance from the observer to the rocket at the moment the rocket is 4,000 feet above the launching pad and is rising at a rate of 880 feet per second. Use the variables in the graph below. [10 points]
7. Suppose that \( f(x) = \frac{1}{3} x^3 + x^2 - 3x - \frac{11}{3} \). [Show your work!] [10 points]

(a) Find the critical numbers of \( f(x) \), then test each critical number for a relative max/min or neither.

(b) Find the open intervals on which \( f(x) \) is increasing, and the open intervals on which \( f(x) \) is decreasing.

(c) Find the open intervals on which the graph of \( f(x) \) is concave up, and the open intervals on which the graph of \( f(x) \) is concave down.

(d) Find the inflection point(s) on the graph of \( f(x) \).

(This problem is continued on the next page.)
(e) Make a sketch of the graph of $f(x)$ on the coordinate system below. Label and give the coordinate of the points where the tangent line is horizontal, and the coordinates of the inflection points.

8. 240 feet of fencing is available to enclose and subdivide a rectangular plot of land. The enclosure is to be subdivided into three parts of equal area (see diagram below). Find the overall dimensions that will maximize the total area of the enclosed plot of land. [Show your work and justify that your dimensions maximize the area.] [10 points]
9. Suppose that $x$ and $y$ are related by the equation $xy + x^2 - y^2 = 4$. [10 points]

(a) Using implicit differentiation, find $\frac{dy}{dx}$.

(b) Find an equation of the tangent line to the graph of $xy + x^2 - y^2 = 4$ at the point $(2, 2)$.

10. Suppose that $f(x, y) = \frac{x^3}{3} + \frac{y^2}{2} - xy$. [10 points]

(a) Find all the critical points of $f(x, y)$.

(b) Classify (relative max/min or saddle point) the critical points found in part (a).
(This page is for scratch work.)

- THE END -
Have a happy and safe Holidays!!