MAT 285          Final Exam          December 16, 2002

Signature:

Instructions: Write the answers and show the main steps of your work on this test sheet. There are 11 questions on 16 pages (including this cover). Be sure you have all 16 pages (8 sheets) and that you do all 11 problems! The Final Exam is scored on a basis of 100 points and will count 20% of your final grade.

PUT YOUR NAME ON THE TOP OF EACH SHEET - NOW!

This Exam has 4 parts corresponding to the 4 major units of the course. You should spend no more than 25 minutes on each part and be sure that you get to the easier problems in each part. Where indicated, you must show your work to get full credit!

DO NOT WRITE ON THE REST OF THIS COVER SHEET!

Part I:

Part II:

Part III:

Part IV:

Total:

Have a nice and safe Holiday season!
Problem 1 (12 points) Circle the correct answer for each part.

a. (3 points) The following table gives the average height $H$, in inches, of a certain population for various ages $A$, in years.

<table>
<thead>
<tr>
<th>Age $A$ (year)</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height $H$ (inches)</td>
<td>63.50</td>
<td>65.85</td>
<td>68.15</td>
<td>70.40</td>
</tr>
</tbody>
</table>

Give the slope (with appropriate units) of a linear equation, age as a function of height, which models these data reasonably well.

2.3 in. per yr.; 2.3 yr. per in.;

0.435 in. per yr.; 0.435 yr. per in.; none of these.

b. (6 points; 3 points each part) It costs $1,250 per week to run a small widget factory (lights, heat, etc.). The raw materials and labor costs of producing one widget is $4.25. Widgets sell for $7.00 each.

(i) The weekly cost in dollars for producing $q$ widgets is:

$C(q) = 7.00q + 1,250; \quad C(q) = 7.00q - 1,250; \quad C(q) = 4.25q - 1,250; \quad C(q) = 4.25q + 1,250; \quad \text{none of these.}$

(ii) The weekly profit in dollars from producing and selling $q$ widgets is:

$P(q) = 7.00q + 1,250; \quad P(q) = 7.00q - 1,250; \quad P(q) = 2.75q - 1,250; \quad P(q) = 2.75q + 1,250; \quad \text{none of these.}$

c. (3 points) The future value after 15 years of an investment account is $500. The account earns 1.75% interest per year, compounded continuously. Find the present value of the account to the nearest cent.
Problem 2 (6 points) The number of weeds $N$ in a garden is proportional to the square root of the area (in square feet) of the garden. Let $k$ denote the proportionality constant. Suppose that for a garden of 1000 square feet in area, there are 86 weeds.

a. (3 points) To 4 decimal places, find $k$:

b. (3 points) Using this model, find the area of a garden that has 125 weeds.
Problem 3 (7 points) A garden shop wants to build a rectangular planter in front of its building. The front wall of the planter is to be constructed from redwood at a cost of $9.50 per linear foot. The remaining three sides will be constructed of concrete block at a cost of $5.75 per linear foot.

\[ \begin{array}{c}
  x & \text{Planter} \\
  & y \\
\text{Redwood Wall} \\
\end{array} \]

a. (2 points) Give the formula for the total cost \( C \) of the walls of the planter in terms of \( x \) and \( y \).

b. (2 points) Give the formula for the area \( A \) of the planter in terms of \( x \) and \( y \).

c. (3 points) Write an equation for the total cost of the walls if the planter is to have an area of 1500 sq. ft. You may give your answer in terms of \( x \) or \( y \) (but not both).
Part II

Problem 4 (13 points) Let \( f(x) = \frac{(6x + 12)}{x^2} \).

[CHECK YOUR INPUT: have you entered the function correctly?]

a. (11 points) On the axes below, sketch the graph of this function. The window is \(-7 \leq x \leq 7\) and \(-3.4 \leq y \leq 3.4\). Label on the graph all local maxima, local minima, and inflection points. Give their coordinates to two decimal places. Label the horizontal and vertical asymptotes. Give their equations.

b. (1 point) At \( x = 55 \) the function is:  
   
   INCREASING  DECREASING  (circle one).

c. (1 point) At \( x = -55 \) the function is concave  
   
   UP  DOWN  (circle one).
Problem 5 (12 points)

a. (3 points) Give the definition of \( f'(x) \). (Hint: Your answer will involved a limit.)

b. For each of the following questions, circle the correct answer.

(i) (3 points) The cost (in dollars) of producing \( x \) screwdrivers is given by the equation \( C(x) = 0.01x^2 - 0.6x + 240 \). At the production level of 60 screwdrivers, the marginal cost is:

- $4 per screwdriver
- 60 cents per screwdriver
- 0.4 screwdriver
- 60 cents equal to the average cost
- none of these

(ii) (3 points)

\[ y = f(x) \]

At which of the following points is \( f(x) \) differentiable (circle your answer(s)): \( p \) \( q \) \( r \)

At which of the following points is \( f(x) \) continuous (circle your answer(s)): \( p \) \( q \) \( r \)
(iii) (3 points)

The graph of $f(x)$ is pictured above. Which of the pictures below is the graph of the derivative of $f(x)$:

A  B  C  None of the below.

A:

B:

C:
Part III

Problem 6 (10 points) For each of the following questions, circle the correct answer.

a. (2 points) The equation \( x^2 - 2xy + y^7 = 15 \) defines \( y \) implicitly as a function of \( x \). Differentiating implicitly, we get:

\[
\frac{dy}{dx} = \frac{-2(x - y)}{2x - 7y^6}; \quad \frac{dy}{dx} = \frac{2x - 7y^6}{2(x - y)}; \quad \frac{dy}{dx} = \frac{-2(3x + y)}{2x + 3y^2};
\]

\[
\frac{dy}{dx} = \frac{2(x - y)}{2x - 7y^6}; \quad \text{none of these.}
\]

b. (3 points) The global maximum of \( f(x) = x^3 - 3x^2 - 24x + 5 \) on the interval \(-3 \leq x \leq 7.1\) occurs at:

\[
x = -2; \quad x = 7; \quad x = \frac{7}{2}; \quad x = 7.1; \quad \text{none of these.}
\]

c. (2 points) The cost function for a product is \( C(x) = x^2 + 10,000 \). The average cost is minimized when:

\[
x = 15; \quad x = 20; \quad x = \frac{7}{2}; \quad x = 100; \quad \text{none of these.}
\]
d. (3 points) A circular oil spill is increasing in size. Recall that the area $A$ of a circle of radius $r$ is given by $A = \pi r^2$. If the area of the spill is increasing at a rate of 16 square miles per day and the radius of the spill is 2 miles, then:

the rate of change of the radius of the spill is \( \frac{4}{\pi} \) square miles per day;

the rate of change of the radius of the spill is \( -\frac{4}{\pi} \) square miles per day;

the rate of change of the radius of the spill is \( \frac{4}{\pi} \) miles per day;

the rate of change of the radius of the spill is \( -\frac{4}{\pi} \) miles per day;

none of the above.
Problem 7 (7 points) The population of a country is modelled by the logistic function

\[ P(t) = \frac{700}{1 + 49e^{-0.02t}}, \]

where \( t \) is the number of years since 1900 and \( P(t) \) is in millions. Give units with answers.

a. (1 point) Find the initial population.

b. (3 points) When is the population growing fastest and at what rate?

c. (3 points) The farmers of the country currently can provide sufficient food for a population of 500,000,000 people. In what year will the population reach 500,000,000 people?
Problem 8 (8 points) A rectangular pasture is to be enclosed with fencing except along the river bank where no fencing will be used. Also, the pasture is to be subdivided into two parts of equal area by a fence that is perpendicular to the river bank (see diagram). 900 yards of fencing are available.

Find the overall dimensions of the pasture that maximizes its area and find the maximum area. Give units. Show work.
Part IV

Problem 9 (9 points) For each of the following questions, circle the correct answer.

a. Let \( z = z(x, y) \) be given. Which of the following statements expresses the inequality: \( \frac{\partial z}{\partial x} \big|_{(x_0, y_0)} < 0 \)?

- At the point \((x_0, y_0)\), \( z \) is increasing in the positive \( x \) direction;
- At the point \((x_0, y_0)\), \( z \) is decreasing in the positive \( x \) direction;
- At the point \((x_0, y_0)\), \( z \) is increasing in the positive \( y \) direction;
- At the point \((x_0, y_0)\), \( z \) is decreasing in the positive \( y \) direction;
- none of these.

b. The Cobb-Douglas Production function for a small printing firm is \( P = 1.25N^{0.6}V^{0.4} \) where \( N \) is the number of workers, \( V \) is the value of the equipment in thousands of dollars and \( P \) is the production level in thousands of pages per day. Suppose that this firm has a labor force of 175 and the value of its equipment is $1,000,000. If the work force is decreased slightly, then production will (circle the correct answer):

- increase at the rate of 1,506 pages per day;
- decrease at the rate of 1,506 pages per day per $1,000 of value of the equipment;
- decrease at the rate of 1,506 pages per day per worker;
- increase at the rate of 1,506 per day;
- remain the same;
- none of these.
c. Consider the function $z_1(x, y) = \frac{2x^2 - y^2}{25}$, pictured below in the 3D window $-10 \leq x \leq 10$; $-10 \leq y \leq 10$; $-5 \leq z \leq 10$.

Circle the letter which corresponds to the slice of this surface for $y = 3$, that is, the cross section perpendicular to the $y$-axis at $y = 3$.

A  B  C  D  E  F
Problem 10 (8 points) Minimize \( f(x, y) = 3x^2 + 2y^2 - 3x \cdot y + 16 \) subject to the constraint \( x + y = 32 \) using the method of Lagrange multipliers. Give the minimum value of \( f(x, y) \) subject to the constraint \( x + y = 32 \)

SHOW THE MAIN STEPS OF YOUR WORK
Problem 11 (8 points) Let \( f(x, y) = x^3 + 3x^2 - 9x + y^2 - 2y + 12 \). Find all local maxima and minima and verify that they are indeed local maxima or minima.

SHOW THE MAIN STEPS OF YOUR WORK