Final Examination MAT 284 Barth and Lewis
Monday, May 9, 2005. 10:15 am – 12:15 pm

Version A

Print name:

Sign name:

Student Identification Number:

Instructor (Barth or Lewis):

TO RECEIVE CREDIT, work and reasoning should be shown for every problem except those indicated by (no work needed).

There are 32 questions (each worth 4 points) on 10 pages.

CHECK THAT YOU HAVE THE COMPLETE TEST!

Do not write below this line

Questions 1 to 9

Question 10

Questions 11 to 21

Question 22

Questions 23 to 32

Total

Examination # 1
(Questions 1 to 9 and 10 and 22)

Examination # 2
(Questions 11 to 21 and 22)

Examination # 3
(Questions 23 to 32 and 10)
1. A company sells a product at $11 per unit. Its fixed cost is $15,000 and its variable cost per unit is $7.

a) Find a function giving the company's profit $P = P(q)$ if it produces and sells $q$ units.

\[ P(q) = \]

b) At what level of production will there be a loss of $3,000?

(Show your work)

2. If \( f(s) = \frac{1}{s^3 - 2} \) and \( g(s) = \sqrt{s + 1} \), then \((f \circ g)(s) = \)

(You need not simplify your answers.)

3. The domain of \( g(z) = \frac{1}{\sqrt{4 - 2z}} \) is

(Show your reasoning)
4. If $10^{\log(x - 3)} = 7$, then $x =$

(Show your work)

5. The supply and demand equations (in some order), with prices in dollars, are: $p = -2q + 48, p = 6q + 8$. If a tax of 40¢ per unit is imposed on the supplier,
   a) write the supply equation
   b) write the demand equation

(No work needed)

6. The cost to produce 20 units is $110 and the cost to produce 10 units is $70. The cost $c = c(q)$ in dollars is a linear function of output $q$, i.e., $c$ and $q$ are linearly related. Find $c = c(q)$.

(Show your work)

7. Express $2\log_x x - 3\log_x(x+1)$ as a single logarithm:

(Show your work)
8. A company has 1,000 units in stock, now selling at $3 per unit. Next month the price will increase by $1. The company wants total revenues from the sale of the units to be no less than $3,600. Let $x$ be the maximum number of units that can be sold this month. Then $x$ must satisfy the following equation or inequality (you need not simplify or solve).

9. If the demand function is $p = 200 - 2q$ and the total cost $c = 300 + 60q$, where $q$ is the number of units, then the profit function is. (Show your reasoning)

10. A company holds a workshop for at least 30 people. If 30 people attend, the charge is $200 each, and the company will reduce the charge for everyone by $4 for each person above 30 who attends. Let $x$ be the number of people who attend. Write the company's total revenue in dollars as a function of $x$. (Here $30 \leq x < 80$. Do not simplify your answer or maximize $R$. (Show your work)

$$R =$$

[If you wish to let $x$ be the number of decreases, you may do so, but check here _______ and write $R$ below].

$$R =$$
11. \[ \frac{d}{dx} (5x^3 + 6) = \] 
(No work needed)

12. Differentiate \( y = x^2 e^{(2x + 1)} \) and do not simplify your answer. (Show your work)

13. Differentiate \( y = \frac{4t + 2}{t^3} \) and do not simplify your answer. (Show your work)

14. Differentiate \( y = \frac{5}{\sqrt{2x^3 + 1}} \) and do not simplify your answer. (Show your work)
15. Find \( \lim_{x \to 4} \frac{x^2 - 2x - 8}{x^2 - 5x + 4} \)

(Show your reasoning)

16. Find the slope of the curve \( y = -8x + x^4 \) at \( x = 1 \)

(Show your work)

17. Let \( q \) be the number of units, let \( p \) be the price per unit in dollars, and let the demand function be \( p = 100 - 2q \). Find the marginal revenue function.

(Show your work)

18. Let \( y = w^3 \) and let \( w = 5 - x^2 \). Use the chain rule to find \( \frac{dy}{dx} \) at \( x = 1 \) (first state the chain rule):

(Show your work)
19. Evaluate \( \lim_{{x \to \infty}} \frac{4 + x - x^2}{2x^3 - x + 1} \).

(No work needed)

20. Differentiate \( y = \ln\left( x^5 \sqrt{1 + x^2} \right) \).

(You do not need to simplify your answer but show your work)

21. Use the DEFINITION of derivative to find the derivative of \( f(x) = x^2 - 1 \).

First state the definition.

(Show your work)

\[
\frac{d}{dx} \left[ e^{\ln(4x^2 + 6)} \right] =
\]

(Show your work.)
23. Let \( y = x^3 + 9x^2 + 24x - 7 \). Find all values of \( x \) for which the curve is concave up.
(Show your work and, if there are none, state “none”)

24. Let \( f(x) = -x^3 + 3x^2 + 24x - 8 \). Find the critical points of \( f(x) \).
(Show your work and, if there are none, state “none”)

25. The curve \( y = -x^3 + 3x^2 + 9x + 8 \) has a critical point at \( x = -1 \). Determine if this point is a relative maximum, relative minimum, or neither.
(Show your work)
26. Let \( y = -x^3 + 15x^2 - 48x + 2 \). Determine whether or not the point \( x = 5 \) is an inflection point. Justify your answer.

\[
\int_{-2}^{2} \frac{2}{x^2} \, dx =
\]
(Show your work)

27. The demand function is \( p = -4q + 400 \), where \( q \) is the number of units and \( p \) is the price per unit in dollars. Find the output at which total revenue is the maximum. (Prove your conclusion)

28. If \( p = 800 - 2q^2 \), where \( q \) is the number of units and \( p \) is the price per unit, then find the (point) elasticity of demand at \( q = 10 \). (Show your work)
30. Suppose that the marginal revenue function is \( 500 - 12q^2 \), where \( q \) is the number of units. Find the demand function. (Show your work)

31. Presently, at $50 per sweater, a company is selling 400 sweaters, while at $40 per sweater it estimates that it will sell 500 sweaters. Find the (approximate) elasticity of demand. (Show your work)

32. Let the total cost function be \( c = q^2 + 3q + 400 \), where \( q \) is the number of units. At what level of output \( q \) will the average cost per unit be at the minimum. [You must show that you are at the minimum.]

There are 32 questions. Be sure that you have done each one of them.