Final Examination    MAT 284    Barth and Fatica
Monday, May 5, 2003.    5:00 pm – 7:00 pm

Version A

Print name:

Sign name:

Student Identification Number:

Instructor (Barth or Fatica):

TO RECEIVE CREDIT, work and reasoning should be shown for every problem except those indicated by (no work needed).

There are 32 questions (each worth 4 points) on 10 pages.

CHECK THAT YOU HAVE THE COMPLETE TEST!

Do not write below this line

Questions 1 to 9  _________  Examination # 1
                     (Questions 1 to 9 and 10 and 22)
Question 10  _________
Questions 11 to 21  _________  Examination # 2
                     (Questions 11 to 21 and 22)
Question 22  _________
Questions 23 to 32  _________  Examination # 3
                     (Questions 23 to 32 and 10)

Total  _________
1. A company sells a product at $11 per unit. Its fixed cost is $15,000 and its variable cost per unit is $7.

   a) Find a function giving the company's profit $P = P(q)$ if it produces and sells $q$ units.

   $P(q) =$

   b) At what level of production will there be a loss of $3,000?

   (Show your work)

2. If $f(s) = \frac{1}{s^3 - 2}$ and $g(s) = \sqrt{s + 1}$, then $(f \circ g)(s) =$

   (You need not simplify your answers.)

3. The domain of $g(z) = \frac{1}{\sqrt{4 - 2z}}$ is

   (Show your reasoning)
4. If \(10^{\log(x - 3)} = 7\), then \(x = \)

(Show your work)

5. The supply and demand equations (in some order), with prices in dollars, are:
\[ p = -2q + 48, \quad p = 6q + 8. \]
If a tax of 40¢ per unit is imposed on the supplier,
\(a)\) write the supply equation

\(b)\) write the demand equation

(No work needed)

6. The cost to produce 20 units is $110 and the cost to produce 10 units is $70. The cost \(c = c(q)\) in dollars is a linear function of output \(q\), i.e., \(c\) and \(q\) are linearly related.
Find \(c = c(q)\).

(Show your work)

7. Express \(2 \log_3 x - 3 \log_3 (x+1)\) as a single logarithm:

(Show your work)
8. A company has 1,000 units in stock, now selling at $3 per unit. Next month the price will increase by $1. The company wants total revenues from the sale of the units to be no less than $3,600. Let \( x \) be the maximum number of units that can be sold this month. Then \( x \) must satisfy the following equation or inequality (you need not simplify or solve).

9. If the demand function is \( p = 200 - 2q \) and the total cost \( c = 300 + 60q \), where \( q \) is the number of units, then the profit function is.
(Show your reasoning)

10. A company holds a workshop for at least 30 people. If 30 people attend, the charge is $200 each, and the company will reduce the charge for everyone by $4 for each person above 30 who attends. Let \( x \) be the number of people who attend. Write the company's total revenue in dollars as a function of \( x \). (Here \( 30 \leq x < 80 \). Do not simplify your answer or maximize \( R \).
(Show your work)

\[
R = 
\]

[If you wish to let \( x \) be the number of decreases, you may do so, but check here _______ and write \( R \) below].

\[
R = 
\]
11. \[ \frac{d}{dx}(5x^3 + 6) = \]
   (No work needed)

12. Differentiate \( y = x^2 e^{(2x + 1)} \) and do not simplify your answer.
   (Show your work)

13. Differentiate \( y = \frac{4t + 2}{t^3} \) and do not simplify your answer. (Show your work)

14. Differentiate \( y = \frac{5}{\sqrt{2x^3 + 1}} \) and do not simplify your answer.
   (Show your work)
15. Find \( \lim_{x \to 4} \frac{x^2 - 2x - 8}{x^2 - 5x + 4} \)

(Show your reasoning)

16. Find the slope of the curve \( y = -8x + x^4 \) at \( x = 1 \)

(Show your work)

17. Let \( q \) be the number of units, let \( p \) be the price per unit in dollars, and let the demand function be \( p = 100 - 2q \). Find the marginal revenue function.

(Show your work)

18. Let \( y = w^3 \) and let \( w = 5 - x^2 \). Use the chain rule to find \( \frac{dy}{dx} \) at \( x = 1 \) (first state the chain rule):

(Show your work)
19. Evaluate \( \lim_{x \to \infty} \frac{4 + x - x^2}{2x^3 - x + 1} \).

(No work needed)

20. Differentiate \( y = \ln \left[ x^5 \sqrt{1 + x^2} \right] \)

(You do not need to simplify your answer but show your work)

21. Use the DEFINITION of derivative to find the derivative of \( f(x) = x^2 - 1 \).
First state the definition.
(Show your work)

22. \( \frac{d}{dx} \left[ e^{\ln(4x^2 + 6)} \right] = \)

(Show your work.)
23. Let \( y = x^3 + 9x^2 + 24x - 7 \). Find all values of \( x \) for which the curve is concave up. (Show your work and, if there are none, state "none")

24. Let \( f(x) = -x^3 + 3x^2 + 24x - 8 \). Find the critical points of \( f(x) \). (Show your work and, if there are none, state "none").

25. The curve \( y = -x^3 + 3x^2 + 9x + 8 \) has a critical point at \( x = -1 \). Determine if this point is a relative maximum, relative minimum, or neither. (Show your work)
26. Let \( y = -x^3 + 15x^2 - 48x + 2 \). Determine whether or not the point \( x = 5 \) is an inflection point. Justify your answer.

27. \[ \int \frac{2}{x^4} \, dx = \]
(Show your work)

28. The demand function is \( p = -4q + 400 \), where \( q \) is the number of units and \( p \) is the price per unit in dollars. Find the output at which total revenue is the maximum.
(Prove your conclusion)

29. If \( p = 800 - 2q^2 \), where \( q \) is the number of units and \( p \) is the price per unit, then find the (point) elasticity of demand at \( q = 10 \).
(Show your work)
30. Suppose that the marginal revenue function is $500 - 12q^2$, where $q$ is the number of units. Find the demand function.
   (Show your work)

31. Presently, at $50 per sweater, a company is selling 400 sweaters, while at $40 per sweater it estimates that it will sell 500 sweaters. Find the (approximate) elasticity of demand.
   (Show your work)

32. Let the total cost function be $c = q^2 + 3q + 400$, where $q$ is the number of units. At what level of output $q$ will the average cost per unit be at the minimum. [You must show that you are at the minimum.]

There are 32 questions. Be sure that you have done each one of them.