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MAT 397
FALL, 2004
Final Exam

INSTRUCTIONS:
(a) Check to see that you have 9 pages and 15 questions. No credit will be given for problems from missing pages.
(b) Do not take the exam apart.
(c) Put your name on every page.
(d) Show all your work. Minimal credit will be given for answers without supporting work.
(e) The number in parentheses by each problem is the point value out of 200 points.

1. (24) Find the indicated partial derivatives (you do not need to simplify your answers):

(a) \( f_x \) for \( f(x, y) = \frac{7xy}{3x^5 - 4y} \).

(b) \( h_{yz} \) for \( h(x, y, z) = x \sin(yz) \).

(c) \( \frac{\partial^2 k}{\partial x^2} \) for \( k(x, y) = (x^2y + 1)^3 \).
2. (8) Find the angle between the planes

\[ x + 2y - z = 1 \quad \text{and} \quad 2x - 3y - 4z = 7. \]

3. (18) Given the function \( f(x, y, z) = xy + yz + zx \) find the following:

(a) The gradient of \( f \) at the point \((2, -1, 0)\).

(b) The directional derivative of \( f \) at the point \((2, -1, 0)\) in the direction of the vector \(< -1, 1, 1 >\).

(c) The direction of fastest increase of \( f \) at the point \((2, -1, 0)\).
4. (10) Find an equation for the tangent plane to the surface $2x^2 + 4y^2 + z^2 = 4$ at the point $P(1, 0, \sqrt{2})$.

5. (12) Consider the parametric curve given by

$$x = 2\sin t, \quad y = 3\cos t, \quad \text{for} \quad 0 \leq t \leq \frac{\pi}{3}$$

(a) Sketch the graph of this curve. Label at least 2 points on the curve with the appropriate values of $x$, $y$, and $t$.

(b) Set up an integral (including the endpoints) which gives the length of this parametric curve. **Do not attempt to evaluate this integral.**
6. (12) Let $r = 1 + \sin \theta$.

(a) Sketch the graph of this curve.

(b) Set up an integral that gives the area inside this curve. Include the limits of integration. Do not attempt to evaluate this integral.

7. (9) Assume that $v(t) = 3e^{-t} \mathbf{i} - 6t \mathbf{j}$ and $\mathbf{r}(0) = \mathbf{i} + \mathbf{j}$. Find $\mathbf{r}(t)$. 
8. (12) (a) Find parametric equations for the line of intersection of the planes
\[ 3x - 2y + z = 5 \quad \text{and} \quad x + 3y - 2z = 4. \]

(b) Give the symmetric equations for the line in part (a).

9. (15) Consider the surface given by the equation \( z = 4r^2 \) in cylindrical coordinates.

(a) Give an equation for this surface in cartesian (that is, \( xyz \)) coordinates.

(b) Give an equation for this surface in spherical coordinates.

(c) Describe this surface.
10. (16) Find all critical points for the function $f(x, y) = x^2 + y^2 + x^2y + 4$. Indicate whether each critical point is a local maximum, a local minimum, or a saddle point.
11. (16) Use Lagrange multipliers to find the maximum and minimum of \( f(x, y, z) = 3x + 15y - 2z \) subject to the constraint \( 3x^2 + 5y^2 + z^2 = 13 \).
12. (12) Use polar coordinates to evaluate
\[ \int_{-2}^{0} \int_{0}^{\sqrt{4-y^2}} (x^2 + y^2)^2 \, dx \, dy. \]

13. (12) Set up an integral in spherical coordinates (including the limits of integration) that gives the volume of the solid in the first octant which lies inside the sphere \( x^2 + y^2 + z^2 = 4 \) and below the cone \( z = \sqrt{3(x^2 + y^2)} \). Do not attempt to evaluate this integral.
14. (12) Set up an integral (including the limits of integration) that gives the area of the part of the surface $z = 4 - y^2$ that is above the $xy$-plane and between the vertical planes $x = -1$ and $x = 1$. Do not attempt to evaluate this integral.

15. (12) Set up a triple integral (including the limits of integration) that gives the volume of the tetrahedron in the first octant bounded by the coordinates planes and the plane $2x + 3y + z = 6$. Do not attempt to evaluate this integral.