Math 397 - Fall 2002  Final Exam

Name: ________________________________  (Please Print.)

SU ID Number: ____________________________

Do all your work on this exam. Unsupported answers may receive little or no credit.

1. (a) Find the area of the triangle in 3-space with vertices $A(2,3,0)$, $B(3,-1,2)$ and $C(4,4,-2)$.

   \[ x = 3 - 2t \]

   (b) Find the point where the line \( y = 4 + t \) \(-\infty < t < \infty\), meets the plane \( x + 2y + z = 6 \).

   \[ z = 1 + 3t \]
2. The position of a particle at time $t$ is given by $r(t) = \sin 2t \, i - \cos 2t \, j + 3t \, k$.

Find the distance traveled between $t = 0$ and $t = 2$. 
3. For the following limit, either evaluate it or show that it does not exist.

$$\lim_{(x,y) \to (0,0)} \frac{\sin(x^2 + y^2)}{x^2 + 2y^2}$$
4. Find the directional derivative of $f(x,y,z) = xze^{3y}$ at the point $P(1,0,2)$ in the direction of $\mathbf{v} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

5. Let $f(x, y) = xe^{y-1} - x^3y^2$.

Find the equation of the tangent plane to the surface $z = f(x, y)$ at the point corresponding to $(2,1)$.
6. (a) Find the local maximums, local minimums and saddle points of the function 
\[ f(x,y) = xy - x^3 - y^2. \]

(b) Use the method of Lagrange multipliers to find the maximum and minimum values of 
\[ f(x,y) = 4x^2 - y \text{ subject to } 4x^2 + y^2 = 1. \]
7. Find the area of the region inside the cardioid $r = 1 + \cos \theta$ and outside the circle $r = \frac{3}{2}$. 
8. Evaluate \( \iiint_S xy \, dV \) where \( S \) is the solid in the first octant bounded by the parabolic cylinder \( z = 4 - x^2 \) and the planes \( z = 0, y = x, \) and \( y = 0 \).

9. Find the surface area of the portion of the sphere \( x^2 + y^2 + z^2 = 16 \) that lies above the plane \( z = \sqrt{3} \).
10. Find the mass of the solid bounded below by $z = 0$, on the sides by the sphere $\rho = 3$, and above by the cone $\phi = \frac{\pi}{6}$ if the density is given by $\delta(x, y, z) = 3z$. 