MAT 194  Final Exam  Spring 2005

May 9, 2005

Do not open this booklet until you are told to do so. Show all work required to solve the problems. Incorrect answers not supported by work will receive no partial credit. You may use a calculator on any part of the exam, but you may not share a calculator with another student. If you have any questions, ask one of the proctors.

Name: _______________________________________________________

Signature: ____________________________________________________

Instructor’s Name (circle one): Fisher  Masingila  Waldron

(Do not write below this line)

Page 1  ________ points (out of 10)

Page 2  ________ points (out of 16)

Page 3  ________ points (out of 22)

Page 4  ________ points (out of 20)

Page 5  ________ points (out of 16)

Page 6  ________ points (out of 16)

TOTAL  ________ points (out of 100)
1. Exercising at Sunshine Health Club costs $2 per visit for members and $4 per visit for non-members. A one-year membership costs $100.

a. (4 pts) Let \( x \) = the number of visits to Sunshine Health Club. Find formulas \( M(x) \) and \( N(x) \) for the cost of exercising there as a member and as a non-member, respectively.

b. (3 pts) After how many visits is a membership the cheapest method of exercising at Sunshine Health Club?

c. (3 pts) On one set of axes, make a sketch that illustrates your solutions in part (a) and part (b); be sure to label your work clearly.
2. a. (4 pts) Suppose that $f(x)$ is linear and that $f(-2) = 3$ and $f(6) = -1$. Find a formula for $f$.

b. (4 pts) Suppose that $g(x)$ is the line perpendicular to $f(x)$ with $y$-intercept $= 1$. Find a formula for $g$.

3. For the piecewise defined function $f(x) = \begin{cases} 1 & -2 \leq x < -1 \\ x^2 & 0 \leq x < 2 \\ x & 2 \leq x < 4 \end{cases}$

a. (4 pts) Graph $f(x)$.

b. (2 pts) Determine the domain of $f(x)$.

c. (2 pts) Determine the range of $f(x)$.  

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Neighbors A and B both go to enjoy their pools at noon the same day. Each holds 12,000 gallons of water when filled to capacity. The amount of water in Pool A \( t \) hours after noon is given by \( A(t) = 6000(1.097)^t \). Pool B, initially filled to capacity, immediately begins leaking water at a rate of 4.2% per hour.

a. (2 pts) Is water being added to or leaking from Pool A? At what rate?

b. (4 pts) Find a formula \( B(t) \) for the amount of water in Pool B \( t \) hours after noon.

c. (2 pts) How much water does each pool contain at 2:30pm? [Round to the nearest gallon.]

d. (6 pts) How long will it take to completely fill Pool A? [Round to the nearest tenth.]

e. (8 pts) When will the pools contain the same amount of water? [Round to the nearest hour.]
5. (8 pts) The population of a certain insect is decreasing exponentially according to the formula $P(t) = 5000e^{-0.22t}$, where $t$ is in years. Determine the half-life of the population. [Round to the nearest hundredth].

6. a. (4 pts) How is the function $f(x) = -2(x + 4)^2 + 18$ related to $y = x^2$ (in terms of shifts, stretches, compressions, etc.)?

b. (2 pts) What is the vertex of $f(x)$?

c. (6 pts) Determine the zeros of $f(x)$. 
7. (8 pts) Find a formula for the trigonometric function below.

8. Let $f(x) = 2x^2 + 1$ and $g(x) = 3 - 2x$.
   
a. (4 pts) Find $f(g(x))$ and simplify your answer.

b. (4 pts) Find $g(f(x))$ and simplify your answer.
9. (8 pts) Find a formula for the polynomial function below.

10. (8 pts) Give a possible formula for a rational function with zeros at \( x = -1 \) and \( x = 3 \), vertical asymptotes at \( x = -4 \) and \( x = 2 \), and a horizontal asymptote of \( y = 5 \).