MAT 194 Final Exam Spring 2003

Name ________________________________________________

Instructor ________________________________________________

Work all problems on the test paper. Use the back side of pages if necessary. Show all your work and do not collaborate with others. No books or notes.

(Do not write below this line)

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Total ____________________
1. A telephone customer is trying to decide between two competing plans. Under plan “A” the customer pays a flat fee of $15 per month plus 12¢ for each call beyond the first 50. Under plan “B” there is a fee of $5.00 and every call costs 15¢. Thus, if the customer makes $x$ calls in a given month, his dollar cost under plan B is $y = 5 + 0.15x$.

(a) Write a formula for the cost under plan A.

(b) How many calls must be made in a month in order for plan A to be the cheaper of the two?
2. Let \( f(x) = 1 + \frac{1}{\sqrt{x-3}} \).

(a) What is the domain of \( f \)?

(b) What is the range of \( f \)?

(c) What is the value of \( f^{-1}(1\frac{1}{2}) \)?
3. In an attempted compromise between supporters of the Fahrenheit and Celsius scales, a U.N. subcommittee proposed a new temperature scale combining features of both. The boiling point of water was assigned as 100 degrees T (Т for U. Thant, a former U.N. Secretary General) and the freezing point as 32 degrees T. Thus, 212° F = 100° T and 32° F = 32° T; while 100° C = 100° T and 0° C = 32° T.

(a) Find the conversion formula from $x$ in ° T to $y$ in ° F.

b) What is the temperature in ° T of a comfortable day (72° F)?
4. Kevin thought about buying a certain item in 1990 when its cost was $2500. He didn’t buy the item then; he waited five years, and by then the price was $3100.

(a) Find a formula for the value of the item as a function of years since 1990 if the price increased exponentially.

(b) What is the growth rate?

(c) If the price continues to grow according to the exponential model, determine in what year the item’s cost will be $5000.
A science class holds a competition to see who can make the best projectile. All projectiles are launched from the ground \((t = 0)\). The winning projectile is in the air for five seconds before reaching its maximum height of 100 feet above the ground and then returning to the earth. The projectile’s height, \(h(t)\), in feet above ground is a quadratic function of time, \(t\), in seconds since blast off.

(a) Sketch the projectile’s height above ground as a function of time.

(b) Find a formula for the function \(h(t)\)

(c) Find the zeros of this function algebraically and give their physical interpretation.
6. Consider the function \( s(t) = 72 + 40\sin\left(\frac{\pi t}{2} + \frac{\pi}{6}\right) \)

(a) Specify the following for \( s(t) \):

(i) Amplitude

(ii) Period

(iii) Midline

(iv) Horizontal Shift

(v) Phase Shift

(b) Sketch the graph of \( s(t) \) for one period.
7. Given a right triangle ABC with right angle C and sides abc, where side a is opposite angle A, side b is opposite angle B and c is the hypotenuse.

(a) Find the length of the other sides and the other angles if angle A is 30 degrees and $b = 2\sqrt{3}$.

(b) Find the slope of the hypotenuse if $c = 25$ and $a = 24$. (Assume side a is horizontal and side b extends vertically upward.)
8. Given \( r(x) = \frac{3(x-1)(x^2 - 4)}{(x^2 - 6x + 9)(x + 4)} \),

a. Find the zeros.

b. Find the y-intercept.

c. Find the asymptote(s)

d. Sketch a graph of the function