MAT 112  Final Exam  Spring 2003

May 5, 2003

Instructions:

Do not open this booklet until you are told to do so. Show all work required to solve the problems. Incorrect answers not supported by work will receive no partial credit. You may use a calculator on any part of the exam, but you may not share a calculator with another student. If you have any questions, ask one of the proctors.

Name: ________________________________

Signature: ______________________________

Instructor’s Name (circle one): Hajduk  Reinhard

<table>
<thead>
<tr>
<th>Part:</th>
<th>Points:</th>
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<tbody>
<tr>
<td>I</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td></td>
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<td>III</td>
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<tr>
<td>TOTAL</td>
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PART I

1. a. (3 points) State the commutative property for addition on the set of integers and illustrate with an example.

b. (2 points) Give one example to show that division is not commutative on the set of real numbers.

c. (3 points) Simplify the following algebraic expression $3x + 5y - 2(4x - 2y)$. Be sure to write out each step and the algebraic properties you use.

d. (3 points) Simplify $| -2^2 - (-5)^3 - (-7)|$

2. Consider the linear function given by the following table:

<table>
<thead>
<tr>
<th>x</th>
<th>y = f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
</tr>
</tbody>
</table>

a. (2 points) Find f(2).
b. (2 points) For which values of \( x \) is \( f(x) = 1 \)?

c. (4 points) Find the equation of this function.

3. Solve for \( x \).

a. (3 points) \( \frac{x}{3} + \frac{2}{3} = 2 \)

b. (3 points) \( -2x + 8 > 0 \)

c. (4 points) \( |2x + 1| = 3 \)

d. (3 points) \( 3x - x + 6 = 2x - 3x + 12 \)
4. (4 points) The following graph was obtained using a CBR system (distance vs. time). Describe as fully as possible how the individual walked to create this graph. You may want to add scales on the x- and y-axes.

5. a. (5 points) Solve the following inequality (you must show all your work)
   \[ x^2 - 16 \leq 2x + 6 \]
If \( f(x) = \frac{1}{5}|x + 5| + 5 \):

b. (4 points) Plot the graph of \( f(x) \).

c. (2 points) Identify the \( x \) intercept(s).

d. (2 points) Identify the \( y \) intercept(s).

6. David bought an annual ski pass for $105. Every time he skied he rented equipment for $6.50. If David skis 32 times this winter, how much will he pay in total to ski for the year?

a. (3 points) Identify the variables in this situation. Which one is the dependent variable and which one is the independent variable?

b. (2 points) Write a function (symbolic rule) that models this situation.
c. (2 points) What are the domain and range of the function in this context?

d. (2 points) Solve the problem.

7. Oliver and Marcus are driving race cars on a race track. Oliver enters the course and quickly speeds up. His velocity (in miles per hour), starting from rest, is given by the function: \( f(t) = t^2 + 5t \), (t in seconds). Marcus, on the other hand, is already on the course and is traveling at 64 mph as Oliver is starting. Marcus is slowly speeding up at 5 mph per second.

a. (2 points) Write an equation that describes Marcus’s velocity per second.

b. (3 points) At what time is Oliver driving as fast as Marcus?

c. (3 points) How fast are they both going at this point in time?
8. A student decided to drop a 16-pound ball and a 1-pound ball off the roof of his dormitory to see which landed first. The student could not find a way to get on the roof, so he decided to throw them directly up to the level of the roof by leaning out of his window. Luckily he was ambidextrous and able to throw them both at exactly 1 p.m. at the same velocity, 32 feet per second, and from the same height, his eye level, which was 48 feet above the ground. Both balls reached exactly the level of the roof. We know that

\[ S(t) = \frac{1}{2} gt^2 + v_o t + s_o, \]

where \( S(t) \) is the height of each ball at elapsed time, \( t \) measured in seconds, \( g = -32 \) ft/sec\(^2\), \( v_o \) is the initial velocity in ft/sec, and \( s_o \) is the initial height in ft.

a. (3 points) How high is the roof?

b. (3 points) Which ball landed first?

c. (3 points) When did the 16-pound ball land?

d. (2 points) When did the 1-pound ball pass his eye level on the way down?
PART III

9. (5 points) A 600-seat movie theater charges $5.50 admission for adults and $2.50 for children. If the theater is full and $1911 is collected, how many adults and how many children are in the audience (you must show all your work)?

10. State with justification whether each of the following systems has one solution, no solution, or infinitely many solutions.
   a. (2 points) \[
   \begin{align*}
   x + 2y &= 4 \\
   3x + 6y &= 5
   \end{align*}
   \]
   b. (2 points) \[
   \begin{align*}
   4x - y &= 1 \\
   2x + 3y &= 1
   \end{align*}
   \]
   c. (2 points) \[
   \begin{align*}
   0.5x - 2y &= 3 \\
   x - 4y &= 6
   \end{align*}
   \]
11. Simplify the following expressions.

a. (2 points) \((x - x)^0\)

b. (2 points) \(\frac{3x^2}{x^3}\)

c. (2 points) \(\sqrt[3]{8} + \sqrt{-8}\)

12. True or False? (Indicate by “T” or “F” at the lines on the right.)

a. (2 points) \(-8\) lies in the domain of the function \(f(x) = \sqrt{x}\). 

b. (2 points) \(\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}\) for all numbers \(a, b \geq 0\).

c. (2 points) \(\sqrt{a} + \sqrt{b} = \sqrt{a + b}\) for all numbers \(a, b \geq 0\).